ON THE AERODYNAMICS OF WINDMILL BLADES

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The optimum twist of a windmill blade is examined on the basis of elementary blade-element theory. For a given wind speed and blade angular velocity, it is shown that the maximum power efficiency is achieved when the blade is twisted according to a program that depends upon the variation of the sectional lift and drag coefficients with angle of attack. Results for a typical airfoil cross-section show that the optimum angle of attack decreases from the maximum-lift-coefficient angle of attack at the blade root to greater than eighty percent of this value at the blade tip.

INTRODUCTION

The energy crisis in the United States has caused a considerable growth of interest in alternative sources of energy in the past few years. Among the several energy sources being explored, wind energy — a form of solar energy — shows much promise in selected areas of the United States where the average wind speeds are high. These areas include the Aleutian Islands, the Columbia River Basin, the Atlantic Coast of the New England states, the southeast boundary of Texas, and the Great Plains area, which includes most of Oklahoma. Estimates of the potential contribution of wind power to the energy needs of the nation vary from as low as five percent to as much as one hundred percent. While the latter figure is suspect, it is clear that in the high-wind-speed regions of the country, wind power can, if properly developed, become a significant energy source. Initial estimates suggest that electrical power can be developed from the wind at a cost of approximately $400 per installed kilowatt as compared with $250 to $400 per installed kilowatt for fossil fuels. While energy storage remains problematic for wind power, it would seem that the environmental benignity and low operating costs of wind power coupled with the growing costs of fossil fuels will make wind power increasingly attractive in the future.

The utilization of the energy in the winds requires the development of devices which convert that energy into more useful forms. This is typically accomplished by first mechanically converting the linear velocity of the wind into a rotational motion by means of a windmill and then converting the rotational energy of the windmill blades into electrical energy by using a generator or alternator. For purposes here, we can thus view the windmill as a mechanical device for extracting some of the kinetic energy of the wind and converting it into the rotational energy of the blade motion. This is accomplished, in detail, by having the blades oriented at some angle to the wind so that the wind blowing past the blades exerts an aerodynamic force on them and thereby causes them to rotate.

The question that naturally then arises is: at what angle to the wind should the blades be set? That is, is there a best angle? This, of course, implies an optimization problem, provided we can decide on some measure of windmill performance. To this end, we shall defined the power efficiency η of a windmill as the ratio of the power developed by the windmill (as a result of the torque exerted on the blades by aerodynamic forces) to the wind power in a streamtube whose cross-sectional area is equal to the swept area of the windmill. The power developed by the windmill as a result of the torque T is Ω T, where Ω is the angular velocity of the windmill blades. The wind power in a channel of area A is $\rho AV^3/2$, where ρ is the air density and V is the wind speed in the direction of the windmill axis at the plane of the windmill blades. Thus

$$\eta = 2\Omega T/\rho A V^3 \quad \textbf{Eq. 1}$$

Hence the optimization problem we wish to explore is: given the wind speed V, the swept area A, and angular velocity Ω, how do we twist the windmill blades in order to achieve the maximum efficiency η ?

The problem we have posed has not yet been solved satisfactorily. While Glauert (1) has shown that the maximum power that can be extracted from a flowing airstream is (9/16) $\rho AC^3/2$, his analysis does not show how one is to shape the windmill blades to transfer as much of this power to the rotational motion of the blades. While Kloeffler and Sitz (2) have shown experimentally that there is an optimum pitch for the windmill blade, their results cannot be used to determine the optimum pitch for a given airfoil section. Although Nilberg (3) has addressed the problem of the optimum airfoil shape and blade twist, his results are suspect. Specifically, Nilberg claims the optimum airfoil shape to be two straight line segments joined at a corner. This cannot be the case in practice as it is well known that such a sharp leading edge and midchord corner both lead to premature stall and large turbulent losses because of separation. Furthermore, Nilberg oversimplifies the aerodynamics of the blade by assuming "the force ... of a deflected airstream upon a good blade should be vertical to the blade chord". This is not the case, especially for angles of attack near stall, and one must use empirical data for the aerodynamic coefficients to determine the force. Thus it is clear that the optimum twist has not yet been theoretically determined.

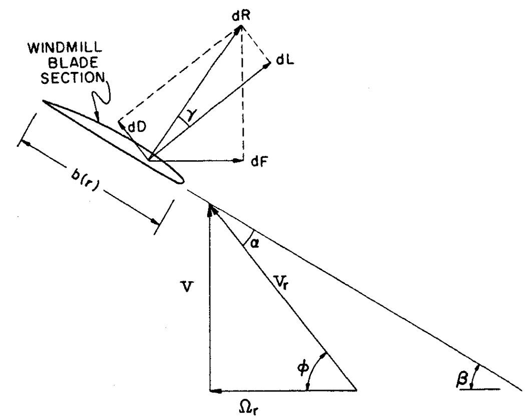

FIGURE 1. Geometry of windmill blade flowfield.

METHOD OF ANALYSIS

To proceed, the efficiency η and the windmill blade shape must be related. To do this, we shall make use of elementary blade-element theory in which each span-wise section of the blade is treated as an airfoil with known sectional lift coefficient C_L and drag coefficient C_D. Thus, the lift force dL and drag force dD acting on an element of the windmill blade of length dr at a distance r from the center are

$$dL = b\,dr\,\tfrac{1}{2}\rho V_r^2 C_L \quad \textbf{Eq. 2}$$

$$dD = b\,dr\,\tfrac{1}{2}\rho V_r^2 C_D \quad \textbf{Eq. 3}$$

Here b(r) is the blade width and V_r is the resultant relative wind speed at the radius r. The resultant relative wind speed V_r has contributions from the wind speed V and the rotational velocity of the blade Ω r. Figure 1 shows a representation of the conditions at a typical radius r. Thus

$$V_r = \sqrt{V^2 + \Omega^2 r^2} \quad \textbf{Eq. 4}$$

It should be noted that V is the wind speed at the plane of the windmill. V is less than the wind speed far ahead of the windmill. Glauert (1) has shown that V is ideally 2/3 of the wind speed far ahead of the windmill. This is true if one ignores the rotational energy in the slipstream downstream of the windmill and any losses due to turbulence of friction.

The inflow angle ϕ is defined as

$$\phi = \tan^{-1}(V/\Omega r) \quad \textbf{Eq. 5}$$

If the angle of attack is α, the blade angle β relative to the plane of rotation then follows as

$$\beta = \phi - \alpha \quad \textbf{Eq. 6}$$

For a given angular velocity Ω and wind speed V, ϕ is a known function of r from Eq. 5. The blade angle β is then given as a function of r by Eq. 6 once the angle of attack α is known. It is the blade angle β that must be known in order to design and construct a windmill blade. For a given Ω and V, we propose to determine α (r) (and then β (r)) from Eq. 6 by taking α to be that function which maximizes η.

The component of the resultant force dR tending to rotate the blade is dF as shown in Figure 1. Thus the torque dT = rdF is given by

$$dT = r\,b\,dr\,\tfrac{1}{2}\rho V^2 (1 + \cot^2\phi)(C_L \sin\phi - C_D \cos\phi) \quad \textbf{7}$$

Adding the contributions from all elements of the windmill blade from r = 0 to r = R, we have for the total torque T

$$T = \tfrac{1}{2}\rho V^2 B \int_0^R \frac{(C_L \sin\phi - C_D \cos\phi)}{\sin^2\phi}\, r\, b\, dr \quad \textbf{Eq. 8}$$

where B is the number of blades and R is the blade radius. Hence our expression for η is

$$\eta = \frac{X}{\pi} B \int_0^1 \frac{(C_L \sin\phi - C_D \cos\phi)}{\sin^2\phi} \left(\frac{b}{R}\right) \xi\, d\xi \quad \textbf{Eq. 9}$$

where $\zeta =$ r/R and X is the tip speed/wind speed ratio,

$$X = \Omega R/V = \cot(\phi_{tip}) \qquad \textbf{Eq. 10}$$

If we fix the number of blades B, the blade width b, radius R, the angular velocity Ω and the wind speed V, η depends only on the variation of angle of attack α with radius since C_L and C_D are functions of α alone for a given airfoil section.

Sketches of C_L as a function of C_D and α for a typical airfoil section (NACA 4424) are given in Figure 2. These results show that C_L increases linearly with α for small α, reaches a maximum, $C_{L_{max}}$ (a typical value being 1.4), at α_{stall}, and decreases thereafter in the so-called stalled region. The drag coefficient C_D increases quadratically with α for small α. As α increases further, so does C_D, although not as α^2.

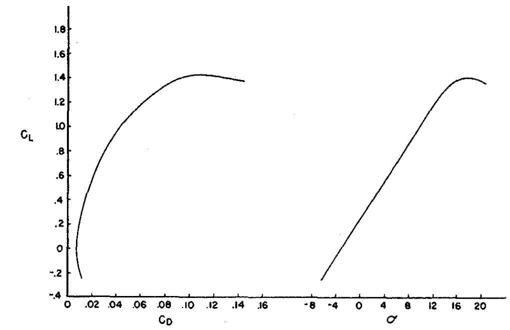

FIGURE 2. Lift coefficient as a function of drag coefficient and angle of attack (NACA 4424 section).

To maximize η, we must then maximize I, defined as

$$I = C_L \sin\phi - C_D \cos\phi \qquad \textbf{Eq. 11}$$

Differentiating this expression with respect to α and setting the result equal to zero, we obtain a maximum when

$$0 = \cos\phi\,(\tan\phi - dC_D/dC_L)\; dC_L/d\alpha \qquad \textbf{Eq. 12}$$

The solution of Eq. 12 is

$$dC_D/dC_L = \tan\phi = V/\Omega r \qquad \textbf{Eq. 13}$$

The other solution, $dC_L/d\alpha = 0$, is included in Eq. 13 since in this case (dC_D/dC_L is infinite and ϕ equals 90º.

Referring to Figure 3, the determination of the optimum angle of attack at any radius r proceeds as follows. First, one finds that point on the C_L versus C_D curve where the angle between the local tangent and the vertical direction equals $\phi = \tan^{-1}(V/\Omega r)$ (the scale of the C_D axis has been exaggerated in Figure 3 by a factor of ten for clarity; thus the tangent of the angle φ' is ten times the tangent of ϕ). This point is labeled A in Figure 3. A horizontal line drawn from point A to the C_L versus α curve then gives point B. A vertical line from point B to the α axis then determines the optimum angle of attack at the radius r. A similar construction at each point along the blade leads to the angle of attack variation with radius that maximizes η.

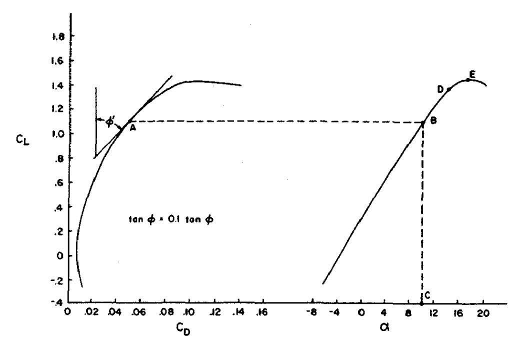

FIGURE 3. Determination of optimum angle of attack at a particular radius r ($r = V/\Omega \tan\phi$).

RESULTS AND DISCUSSION

A typical result for the variation of the optimum angle of attack with radius is shown in Figure 4 for the NACA 4424 airfoil section for tip wind speed ratios X equal to 6, 4 and 2. The variation of angle of attack for X equal to six corresponds to the range of lift coefficients between points D and E in Figure 3 and corresponds to an angle of attack variation from 17.4º at the root to 14.3 at the tip. The optimum blade angle β_{opt} then follows as

$$\beta_{opt} = \tan^{-1}(V/\Omega r) - \alpha_{opt} \qquad \textbf{Eq. 14}$$

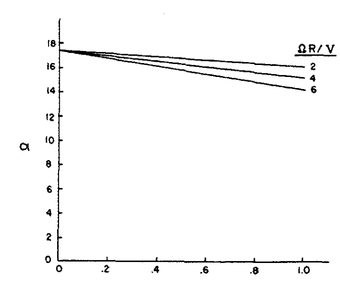

FIGURE 4. Optimum angle of attack distribution (NACA 4424 section).

If X = Ω R/V changes, the inflow angle φ and the optimum angle of attack α_{opt} change. Maintaining β and β_{opt} for difficult values of X would be quite difficult practically as it would require a blade whose twist would vary with X. Thus, in practice, one would most likely optimize β for a particular ratio of tip speed to wind speed and accept non-optimum β for other conditions.

For other airfoil sections, one requires data for C_L versus C_D and C_L versus α to determine the optimum angle. These data can usually be obtained in the literature (e.g. ref. 4). If these data are not available, wind tunnel experiments would be required as the usual linearized aerodynamic theories are inadequate in the stalled region where, as Figure 3 shows, the optimum angles of attack typically lie.

Elementary blade-element theory makes use of isolated airfoil section data for C_L versus C_D and C_L versus α and in doing so assumes that there is no interference between the blades of the windmill. Such an approximation is valid if the windmill solidity (defined as the ratio of the total blade area to the swept area of the windmill) does not exceed approximately 0.1. If the solidity exceeds 0.1, the maximum lift coefficient decreases—in some cases, significantly. Indeed, for a solidity of 0.40, the lift coefficient can decrease by as much as forty-five percent (5). Thus the present calculations are valid for windmills of low solidity, typical of proposed power-generating windmill designs.

ACKNOWLEDGMENT

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